Give 15 minutes

Quiz 8, Linear

11:32
11:34

Name: Key

1. (5 points) Find the coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ of $\mathbf{x}=\begin{bmatrix} 8 \\ 9 \\ -4 \end{bmatrix}$ relative to the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} \right\}.$$

$x = c_1 b_1 + c_2 b_2 + c_3 b_3$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ -1 & 4 & -2 & 9 \\ -3 & 9 & 4 & -4 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 1 & 0 & 17 \\ 0 & 0 & 10 & 20 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 0 & 4 \\ 0 & 1 & 0 & 17 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 55 \\ 0 & 1 & 0 & 17 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\begin{bmatrix} 55 \\ 17 \\ 2 \end{bmatrix} = [x]_{\mathcal{B}}$$

12:09

and the I.M.T

2. (5 points) Suppose $\mathbb{R}^4 = \text{Span}\,\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. *Use the definition of basis* to explain why $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$ is a basis for $\mathbb{R}^4$.

A basis of $\mathbb{R}^4$ must span $\mathbb{R}^4$ and also be linearly independent

Since $\mathbb{R}^4 = \text{Span}\{v_1, \ldots, v_4\}$, $\{v_1, \ldots, v_4\}$ spans $\mathbb{R}^4$.

If $\{v_1, \ldots, v_4\}$ were linearly dependent, the matrix $[v_1\ v_2\ v_3\ v_4]$ would not be invertible and by I.M.T. $\{v_1, \ldots, v_4\}$ would not span

Then the matrix $[v_1\ v_2\ v_3\ v_4]$ spans $\mathbb{R}^4$, so by I.M.T. the columns are lin. independent.